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$$\therefore\ y_{m+1}-y_m=\frac{a}{n(r-s)}\{\sqrt[3]{n^2[(m+1)^3r^2+(n-m-1)s^3]}-\sqrt[3]{n^2[mr^3+(n-m)s^3]}\},$$

which is the general value for the respective altitudes of the n equal parts of the given frustum.

The limits of m are *zero* and n.

$$y_0=x=\frac{as}{r-s},\text{ and } y_n=x+a=\frac{ar}{r-s}.$$

Hence as the altitudes of the equal parts diminish from s to r, $y_1-y_0=$ the greatest altitude and $y_n-y_{n-1}=$the least altitude.

The radius of the mth section is $z_m=\frac{1}{n}\sqrt[3]{n^2[mr^3+(n-m)s^3]}$.

Put $a=12$, $r=3$, $s=2$, and $n=4$.

Then $y_{m+1}-y_m=6\{\sqrt[3]{[2(19m+51)]}-\sqrt[3]{[2(19m+32)]}\}$.

Whence $y_1-y_0=6[\sqrt[3]{(102)}-4]=4.034$; $y_2-y_1=6[\sqrt[3]{(140)}-\sqrt[3]{(102}]=3.121$; $y_3-y_2=6[\sqrt[3]{(178)}-\sqrt[3]{(140)}]=2.596$; and $y_4-y_3=6[6-\sqrt[3]{(178)}]=2.249$.

Also $z_1=\frac{1}{2}\sqrt[3]{(102)}=2.336$; $z_2=\frac{1}{2}\sqrt[3]{(140)}=2.596$; and $z_3=\frac{1}{2}\sqrt[3]{(178)}=2.812$.

Also solved in a very excellent manner by *G. B. M. ZERR*, and *J. SCHEFFER*.

ALGEBRA.

107. **Proposed by CHARLES E. MYERS, Canton, Ohio.**

Given $xyz=18$....(1); $x^2+y^2+z^2=33$....(2); and $(x^2-yz)^3+(y^2-xz)^3+(z^2-xy)^3-3(x^2-yz)(y^2-xz)(z^2-xy)=6561$....(3); to find x, y, and z.

I. **Solution by M. A. GRUBER, A. M., War Department, Washington, D. C.**

Expanding, uniting terms, and extracting square root of [3], we have

$$x^3+y^3+z^3-3xyz=81\text{....[4]}.$$

Substituting $xyz=18$, and transposing, [4] becomes

$$x^3+y^3+z^3=135\text{....[5]}.$$

Put $y=x+v$, and $z=x-v$.

Then, [1] becomes $x^3-xv^2=18$....[6],

[2] becomes $3x^2+2v^2=33$....[7],

and [5] becomes $3x^3+6xv^2=135$ ...[8].

From [6] and [8], we readily find $x=3$. Whence $v=\pm\sqrt{3}$.

$\therefore$ $x=3$, $y=3\pm\sqrt{3}$, and $z=\mp\sqrt{3}$.

II. Solution by HARRY S. VANDIVER, Bala, Pa.

$$xyz=a=18\ldots.(1).$$

$$x^2+y^2+z^2=b=33\ldots.(2).$$

$$(x^2-yz)^3+(y^2-xz)^3+(z^2-xy)^3-3(x^2-yz)(y^2-xz)(z^2-xy)=c=6561\ \ldots.\ (3).$$

Factoring (3) and using the customary notation for symmetric functions, we have

$$(\Sigma x)^2(\Sigma x^2-\Sigma xy)^2=\pm c$$

$$\text{whence } (\Sigma x)(\Sigma x^2-\Sigma xy)=\pm\sqrt{c}\ldots.(4).$$

Now assume that x, y and z are the roots of the cubic

$$v^3-mv^2+nv-p=0\ldots.(5),$$

and we are to determine m, n and p. From (1) we have $p=a$. Also we have identically,

$$(\Sigma x)^2=\Sigma x^2-2\Sigma xy\ldots.(5).$$

Then from (2), (4) and (5) we have the following equations to determine m and n:

$$m(b-n)=\pm\sqrt{c},\quad m^2=b-2n.$$

Eliminating and substituting, we find that the original system is equivalent to

$$\begin{cases} m^3+bm\mp 2\sqrt{c}=0\ldots.(6) \\ v^3-mv^2+[b\mp(\sqrt{c}/m)]v-a=0\ldots.(7) \end{cases}$$

where $v=x$, y or z, and $\sqrt{c}$ has the same sign in both equations.

Of course, this system may be algebraically solved only in special cases, but substituting the given numerical values we obtain, writing (6) and (7) as two systems,

$$\begin{cases} m^3+33m+162=0. \\ v^3-mv^2+[33+(81/m)]v-18=0. \end{cases}$$

$$\begin{cases} m^3+33m-162=0. \\ v^3-mv^2+[33-(81/m)]v-18=0. \end{cases}$$

Both systems admit of being solved by Cardan's Method, giving 9 values for v in each—1 real and 8 imaginary.

Also solved by *H. W. KEATING*, Pittsburg, Pa., *ELMER SCHUYLER*, *J. SCHEFFER*, *G. B. M. ZEER*, and *COOPER D. SCHMITT*.